

AD 606593

1

ANALYTICAL APPROXIMATIONS

Volume 26

Cecil Hastings, Jr.
Elaine Hastings

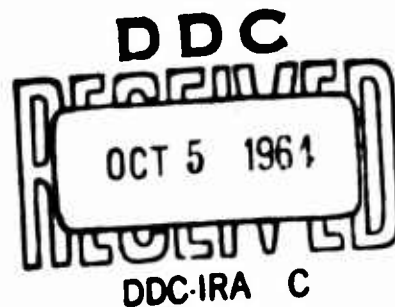
✓ P-1217
Bar

15 November 1957

Approved for OTS release

COPY	1	OF	1
HARD COPY	\$. 1.00		
MICROFICHE	\$. 0.50		

3p



The RAND Corporation

1700 MAIN ST. • SANTA MONICA • CALIFORNIA

COPYRIGHT, 1957
THE RAND CORPORATION

10-26-57

Analytical Approximation

Incomplete Elliptic Integral: To better than .0000,0003 over $45^\circ \leq \phi \leq 90^\circ$ and $45^\circ \leq \alpha \leq 90^\circ$,

$$F(\phi, \alpha) = \int_0^\phi \frac{d\phi}{\sqrt{1 - \sin^2 \alpha \sin^2 \phi}}$$

$$= A(\alpha) + B(\phi, \alpha) \sqrt{1 - \sin^2 \alpha \sin^2 \phi}$$

$$+ C(\alpha) \ln \left[\frac{1}{\cos \phi \sin \alpha + \sqrt{1 - \sin^2 \alpha \sin^2 \phi}} \right] .$$

$$A(\alpha) = 1.3862,9436$$

$$C(\alpha) = 1.0$$

$$+.0965,7350(\frac{\pi}{2} - \alpha)^2$$

$$+.2499,9980(\frac{\pi}{2} - \alpha)^2$$

$$-.0013,0437(\frac{\pi}{2} - \alpha)^4$$

$$+.0572,9764(\frac{\pi}{2} - \alpha)^4$$

$$-.0013,7074(\frac{\pi}{2} - \alpha)^6$$

$$+.0149,5866(\frac{\pi}{2} - \alpha)^6$$

$$-.0002,7227(\frac{\pi}{2} - \alpha)^8$$

$$+.0047,0849(\frac{\pi}{2} - \alpha)^8$$

$$-.0000,9495(\frac{\pi}{2} - \alpha)^{10}$$

$$+.0008,2724(\frac{\pi}{2} - \alpha)^{10}$$

$$+.0010,8108(\frac{\pi}{2} - \alpha)^{12}$$

Analytical Approximation

$$B(\phi, \alpha) = f_1(\phi)g_1(\alpha) + f_2(\phi)g_2(\alpha) + f_3(\phi)g_3(\alpha)$$

$$\begin{aligned} f_1(\phi) = & -.3277,5088 \left(\frac{\pi}{4} - \phi\right)^3 \\ & -.0638,1682 \left(\frac{\pi}{4} - \phi\right)^5 \\ & -.0080,8686 \left(\frac{\pi}{4} - \phi\right)^7 \\ & -.0008,5024 \left(\frac{\pi}{4} - \phi\right)^9 \\ & -.0000,9551 \left(\frac{\pi}{4} - \phi\right)^9 \\ g_1(\alpha) = & .7674,7222 \\ & +.3184,7443 \left(\frac{\pi}{4} - \alpha\right)^2 \\ & +.0820,7991 \left(\frac{\pi}{4} - \alpha\right)^4 \\ & +.0179,8074 \left(\frac{\pi}{4} - \alpha\right)^6 \\ & +.0034,7561 \left(\frac{\pi}{4} - \alpha\right)^8 \\ & +.0013,5976 \left(\frac{\pi}{4} - \alpha\right)^{10} \end{aligned}$$

$$\begin{aligned} f_2(\phi) = & .0298,811 \left(\frac{\pi}{4} - \phi\right)^3 \\ & -.0606,009 \left(\frac{\pi}{4} - \phi\right)^5 \\ & -.0206,017 \left(\frac{\pi}{4} - \phi\right)^7 \\ & -.0040,129 \left(\frac{\pi}{4} - \phi\right)^9 \\ & -.0007,920 \left(\frac{\pi}{4} - \phi\right)^9 \\ g_2(\alpha) = & .0516,668 \\ & -.1689,896 \left(\frac{\pi}{4} - \alpha\right)^2 \\ & -.0998,444 \left(\frac{\pi}{4} - \alpha\right)^4 \\ & -.0344,697 \left(\frac{\pi}{4} - \alpha\right)^6 \\ & -.0065,684 \left(\frac{\pi}{4} - \alpha\right)^8 \\ & -.0043,974 \left(\frac{\pi}{4} - \alpha\right)^{10} \end{aligned}$$

$$\begin{aligned} f_3(\phi) = & -.000727 \left(\frac{\pi}{4} - \phi\right)^3 \\ & +.004444 \left(\frac{\pi}{4} - \phi\right)^5 \\ & -.003986 \left(\frac{\pi}{4} - \phi\right)^7 \\ & -.002520 \left(\frac{\pi}{4} - \phi\right)^9 \\ & -.000621 \left(\frac{\pi}{4} - \phi\right)^9 \\ g_3(\alpha) = & .005752 \\ & -.075208 \left(\frac{\pi}{4} - \alpha\right)^2 \\ & +.073873 \left(\frac{\pi}{4} - \alpha\right)^4 \\ & +.088197 \left(\frac{\pi}{4} - \alpha\right)^6 \\ & -.014601 \left(\frac{\pi}{4} - \alpha\right)^8 \\ & +.044161 \left(\frac{\pi}{4} - \alpha\right)^{10} \end{aligned}$$

Cecil Hastings, Jr.
Elaine Hastings
Copyright 1957
The RAND Corporation